# Efficiency Improvement of PM Disc Motor Using Genetic Algorithm

Goga Cvetkovski[1], Lidija Petkovska[1] and Sinclair Gair[2]

[1] Ss. Cyril & Methodius University, Faculty of Electrical Engineering, P.O. Box 574, 1000 Skopje, Macedonia
Phone: +389 2 3099 143, Fax: +389 2 3064 262, e-mail: gogacvet@etf.ukim.edu.mk, lidijap@etf.ukim.edu.mk

[2] University of Strathclyde, Department of Electronic and Electrical Engineering
204 George Street, Glasgow G1 1XW, Scotland, UK
Phone: +44 141 548 4651, Fax: +44 141 548 4872, e-mail: s.gair@eee.strath.ac.uk

**Abstract** - Design optimisation of electrical machines, in particular permanent magnet synchronous disc motors, is very important, but quite a complicated problem. A reasonably simplified form of the design procedure may be attacked by various approaches accumulated into two main topics; classical optimisation techniques (deterministic methods) or genetic algorithms (stochastic methods). Genetic algorithm (GA) has been used as the most suitable optimisation method for this task. In this paper an optimal design of permanent magnet disc motor using GA as an optimisation tool is performed. At the end, based on results of the FEM calculation of the magnetic field, a comparison of the prototype and the GA optimisation solution is also performed.

**Key words:** permanent magnet disc motor, genetic algorithm, efficiency, optimisation.

## 1. Introduction

The problem of design optimisation of motors sometimes arise because of the stiff competition among manufacturers to produce a motor giving the same performance, but at a reduced cost. Sometimes the application requires a motor of certain weight or shape that has to satisfy certain requirements. Design optimisation of electrical machines, in particular permanent magnet disc motors, is very important, but quite a complicated problem. In general the optimal design of electrical machines is a complex multi-variable, non-linear and constrained optimisation procedure. The non-linear nature of the active materials, together with the discreteness of some design parameters, renders the task of optimisation a mixed real number programming problem. A reasonably simplified form of the design procedure may be attacked by various approaches accumulated into two main topics: classical optimisation techniques (deterministic methods) versus genetic algorithm (stochastic methods). Researchers have used classical (usually gradient based) optimisation techniques for this task for a long time. However, recently, evolutionary computation techniques such as Genetic Algorithm (GA) have been used for optimisation procedures. These methods are claimed to be more successful in converging to a global maximum/minimum avoiding the local ones. Also, they avoid the problem of starting the search from a suitable feasible solution, often encountered in classical optimisation techniques. In this study, the authors of the paper intended to investigate whether this claims are valid for the problem of design optimisation of a permanent magnet disc motor.

## 2. Optimisation design model

The optimisation problem discussed in this study could be presented in general as a constrained optimisation problem which is defined with the following standard form:

$$\begin{aligned} &\text{maximise:} && f(x) \\ &\text{subject to:} && g_i(x) = 0 \quad i=1,2,...m \\ & && h_j(x) < 0 \quad j=1,2,...n \end{aligned} \tag{1}$$

where:

$f(x)$ - efficiency of the motor,
$g_i(x)$ and $h_j(x)$ - equality and inequality constraints.

The efficiency of the motor is selected as an objective function of the optimisation, because the energy saving is especially important in the application of the motor as a drive of an electric vehicle (EV). The permanent magnet disc motor is supplied with energy from a great number of rechargeable batteries on board the vehicle and therefore an improvement of the efficiency is essential.

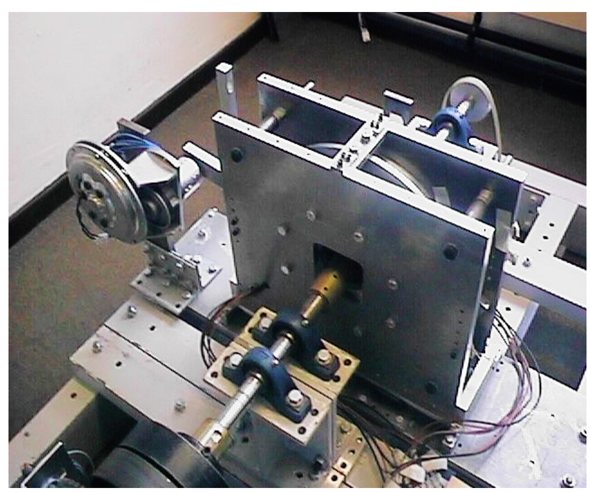

Fig. 1. PMDM prototype view

The general aim of the permanent magnet disc motor (PMDM) optimal design is to obtain a motor with maximised efficiency while satisfying certain performance, magnetic and geometric constraints. The design optimisation is performed on a designed by hand prototype permanent magnet disc motor with rated torque 54 Nm and speed 750 rpm. The permanent magnet disc motor is a double sided axial field motor with two laminated stators having 36 slots and a centred rotor having 8 skewed neodymium-iron-boron permanent magnets with $B_r$=1.17 T and $H_c$=-883 kA/m. A side view of the prototype motor is shown in Figure 1.

## 2. GA Optimisation Method Description

Since John Holland [1] presented the GA as a computer algorithm, a wide range of applications of GA has appeared in various scientific areas, and GA has been proved powerful enough to solve the complicated problems, especially the optimal design problems. GAs are evolutionary search algorithms based on the mechanics of natural selection and natural genetics. GAs implement, in the most simplistic way, the concept of survival of the fittest. The reproductive success of a solution is directly tied to the fitness value it is assigned during evaluation. In this stochastic process, the least-fit solution has a small chance at reproduction while the most-fit solution has a greater chance of reproduction. The search starts from a randomly created population representing the chromosomes and obtains optimum after a certain number of generations of genetic operations. The optimisation is based on the survival of the string structures from one generation to the next, where a new improved generation is created by using the bits of information-*genes* of the survivors of the previous generation.

The created optimal design programme GA-ODEM (Genetic Algorithm for Optimal Design of Electrical Machines) is using the Genetic Algorithm as an optimisation tool [2]. The design variables are presented as vectors of floating-point numbers. The search starts from a randomly created population of strings representing the chromosomes and obtains optimum after a certain number of generations by applying genetic operations. The search can continue indefinitely. Therefore, a stopping rule is necessary to tell the algorithm when it is time to stop. This is achieved in many different ways and is also a user's and a problem dependent. Some of the possible methods are to fix the number of generations and to use the best individual of all generations as the optimum result; to fix the time elapsed and to select the optimum similarly; or to let the entire population converge in to an average fitness with some error margin. The stopping rule applied in this GA optimal design programme is the number of generations.

The parameters of the GA shape the way the algorithm runs. They could be grouped in two groups such as: primary and secondary parameters. There is one primary parameter:

- $N$ the population size which is the number of chromosomes in the population;

and two secondary parameters that define the occurrence probabilities of the GA operators:

- $p_c$ crossover probability;
- $p_m$ mutation probability.

The values that are assigned to all of them are user and problem dependent. In order to make a proper selection of the $p_c$ and $p_m$ value, a very complex and detailed analysis of these parameters and their influence on the quality of the GA search, has been performed. The considered values for this optimal design problem are presented in Table I.

Table I. Values of GA Parameters

| GA Parameters | Value |
|---|---|
| Population size | 20 |
| Number of generations | 15000 |
| Crossover probability | 0.85 |
| Mutation probability | 0.07 |

The main genetic operators of the genetic algorithm in general are reproduction, crossover and mutation.

*A. Reproduction*

Working on the entire population, the reproduction operator creates a new generation from the old generation. Based on the fitness measure of an individual and the average fitness of the population, the reproduction operator, determines the number of copies that particular individual will have in the next generation. The underlying idea in designing the reproduction operator is to give the individual with higher fitness a better chance to be represented in the next generation but leaving the decision to a random variable. The reproduction procedure that is implemented in the optimisation programme is performed by a linear search through a roulette wheel with slots weighted in proportion to string fitness values.

*B. Crossover*

A central feature of genetic algorithms that creates a new chromosome from two "parents" is crossover. Corresponding to biological crossover, the software version combines a pair of parents by randomly selecting a point at which pieces of the parents' vectors of numbers are swapped. Instead of using the simple crossover the swapping is done with the so called arithmetical crossover which is defined as a linear combination of two vectors $x_1$ and $x_2$, after which the resulting offspring is:

$$\mathbf{x}_1' = \mathrm{c} \cdot \mathbf{x}_1 + (1-c) \cdot \mathbf{x}_2 \tag{2}$$

$$\mathbf{x}_2' = \mathrm{c} \cdot \mathbf{x}_2 + (1-c) \cdot \mathbf{x}_1 \tag{3}$$

In the previous equations $c$ could be any number between 0 and 1 or it can be taken as a fixed number; in this case it was adopted to be equal to 0.5. This type of crossover is called uniform arithmetical crossover and with its usage it is guaranteed that the values of the new parents will always be in the domain.

*C. Mutation*

Another step in reproduction is mutation, which involves the random real number generation of a selected variable in its upper and lower bound domain, of the new population. The primary purpose of mutation is to introduce variation into a population. This process is carried out randomly and it is done at a randomly selected place.

Another procedure that is implemented in the optimal design programme is the fitness scaling which improves the overall performance and leads towards better reliability of the GA search.

*D. Linear Scaling*

Linear scaling adjusts the fitness values of all chromosomes such that the best chromosome gets a fixed number of expected offspring and thus prevent it from reproducing too many. The linear scaling method can be presented with the following equation:

$$f_k^{'} = a \cdot f_k + b \tag{4}$$

The coefficients $a$ and $b$ may be chosen in a numerous ways or can be defined as:

$$a = \frac{(C - 1)\, f_{avg}}{f_{\max} - f_{avg}} \qquad b = \frac{f_{avg}(f_{\max} - C f_{avg})}{f_{\max} - f_{avg}} \tag{5}$$

where C is a constant that can be C=1.2-2.0 and the values $f_{max}$ and $f_{avg}$ are the maximum and average value of the fitness for each generation.

After the operators perform their functions, the new generation is produced of members, which have gained new information through the exchange between pairs. The better traits of the "parent" chromosomes are carried along to the future generations. The optimal solution of the PMDM is selected as the best solution of the GA search.

## 3. GA Optimal Design of PMDM

According to the design characteristics of PMDM, some of the parameters are chosen to be constant and some to be variable, such as: inside radius of the PM and the stator cores $\boldsymbol{R_i}$, outside radius of the PM and of the stator cores $\boldsymbol{R_o}$, air-gap **g**, single wire diameter $\boldsymbol{d_w}$ and sloth width $\boldsymbol{b_s}$. Some of these parameters are presented in Figure 2. In the performed optimal design the rotor parameters are considered to be constant, due to the complicated and expensive manufacturing procedure of the permanent magnets.

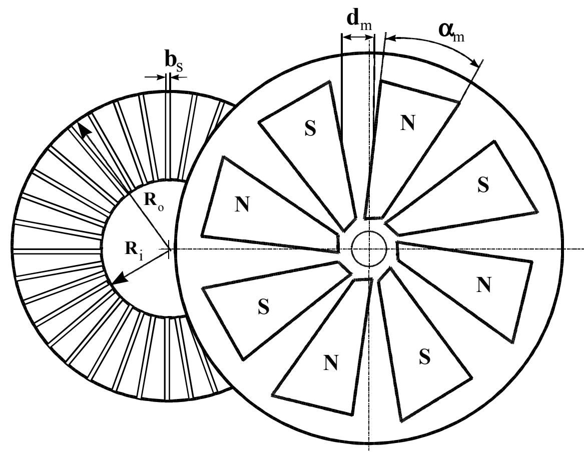


Fig. 2. PMDM parameter presentation

The efficiency of the motor, as an objective function of the optimisation can be presented with the following equation:

$$efficiency = \frac{T \cdot \omega_m}{T \cdot \omega_m + P_{Cu} + P_{Fe} + P_s} \tag{6}$$

where: $T$-rated torque, $\omega_m$-rated speed, $P_{Cu}$-ohmic power losses, $P_{Fe}$-core losses and $P_s$-other constant losses calculated from no load test of the machine. The optimal design process of the permanent magnet disc motor is a maximisation problem of the objective function, where the torque is one of the constraints.

Due to the fact that the optimised motor is a novel type of motor it is important at this stage to define some of the motor parameters and their dependence on the optimisation variables. Therefore the ohmic power losses $P_{cu}$ can be described with the following equation:

$$P_{cu} = N_{ph} \cdot I_{ph}^2 \cdot R_{ph} \tag{7}$$

and the phase resistance with the equation:

$$R_{ph} = \frac{N_{st} \cdot \rho \cdot W_s \cdot N_{sp}\left[2(R_n - R_v) + 1.65(\tau_{ci} + \tau_{co})\right]}{n \cdot S_w} \tag{8}$$

where:
$N_{ph}$ - number of phases
$I_{ph}$ - phase current
$R_{ph}$ - phase resistance
$N_{sp}$ - slots per phase
$N_{st}$ - number of stators
$W_s$ - number of turns per coil
$\tau_{ci}$ - coil pitch on the inside radius
$\tau_{co}$ - coil pitch on the outside radius
$n$ - number of strands in parallel
$S_w$ - cross-section area of the wire

The core losses $P_{Fe}$ can be described with the equation:

$$P_{Fe} = N_{st} \rho_{fe} \left( V_{st} \Gamma_{B_{mst},f} \left( \frac{B_{st}}{B_{mst}} \right)^2 + V_{bi} \Gamma_{B_{mbi},f} \left( \frac{B_{bi}}{B_{mbi}} \right)^2 \right) \tag{9}$$

where:
$\rho_{bi}$ - stator steel mass density
$\Gamma_{Bm,f}$ - stator steel core loss density vs. flux density and frequency
$B_{bi}$ - stator back flux density
$B_{st}$ - stator teeth flux density
$V_{bi}$ - stator back iron volume
$V_{st}$ - stator teeth volume

The volumes of the stator back iron and stator teeth are defined with equation (10):

$$
\begin{aligned}
V_{bi} &= k_{st} \cdot \pi \cdot \left(R_o^2 - R_i^2\right) \cdot w_{bi} \\
V_{st} &= k_{st} \cdot \pi \cdot \left(R_o^2 - R_i^2\right) \cdot d_s - Z\left(R_o - R_i\right) \cdot d_s \cdot b_s
\end{aligned}
\tag{10}
$$

where:
$k_{st}$ - lamination stacking factor
$w_{bi}$ - back iron width
$d_s$ - total slot depth
$b_s$ - total slot width
$Z$ - number of stator slots

On the other hand the values of the flux densities in the back iron and stator teeth are defined with the following equations:

$$B_{st} = \frac{\Phi_\delta}{N_{sm} \cdot S_{st} \cdot k_{st}} \tag{11}$$

$$B_{bi} = \frac{\Phi_\delta}{2 \cdot w_{bi} \cdot \left(R_o - R_i\right) \cdot k_{st}} \tag{12}$$

where:
$\Phi_\delta$ - air gap flux
$N_{sm}$ - number of slots per pole
$S_{st}$ - stator tooth area

These are only part of the equations representing the complex mathematical model of the motor [2] developed for the optimisation analysis. This mathematical model of the motor is implemented in the optimisation procedure of the genetic algorithm that was previously presented.

The optimal solution of the PMDM is selected as the best solution of the GA search. The algorithm created for optimal design of the PMDM [3,4] uses real number representation, arithmetic crossover and linear fitness scaling as an improvement and reliability of the GA search. The lower and upper bound as well as the efficiency results and the optimisation parameters values of the optimisation procedure, in relation to the prototype model, are presented in Table 1.

Table II. Optimisation results

| Parameters | Lower bound | Upper bound | Prototype | GA solution |
|---|---|---|---|---|
| $R_i$ [m] | 0.070 | 0.074 | 0.072 | 0.070 |
| $R_o$ [m] | 0.128 | 0.138 | 0.133 | 0.138 |
| $g$ [m] | 0.0018 | 0.0022 | 0.002 | 0.0018 |
| $d_w$ [m] | 0.001 | 0.0014 | 0.001 | 0.0014 |
| $b_s$ [m] | 0.007 | 0.009 | 0.008 | 0.0084 |
| *efficiency* | - | - | 0.8319 | 0.8621 |

From the results shown in Table II, it is evident that the GA solution of the motor has better parameters compared to the prototype PM disc motor. Some of the specific parameters of the GA optimal solution and the prototype are shown in Table III.

Table III. Prototype and GA solution geometrical data

| Parameters | Description | Prototype | GA solution |
|---|---|---|---|
| $h_s$ [m] | slot height | 0.016 | 0.0234 |
| $h_{sbi}$ [m] | stator back iron length | 0.014 | 0.0147 |
| $b_{thi}$ [m] | stator teeth width on Ri | 0.004566 | 0.003765 |
| $W_s$ [turns] | # of turns/coil | 13 | 11 |

The convergence of the value of the objective function from the GA optimisation search for 15000 generations is shown in Figure 3.

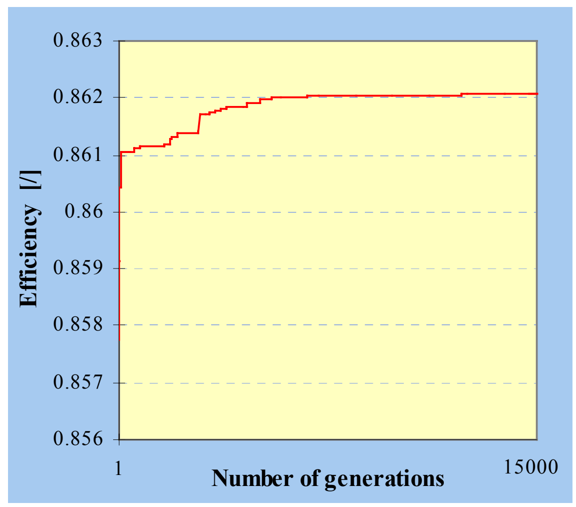


Fig. 3. GA search efficiency change during generations

The changes of the value of the five variables from the GA optimisation search for 15000 generations are presented in Figure 4 and Figure 5.

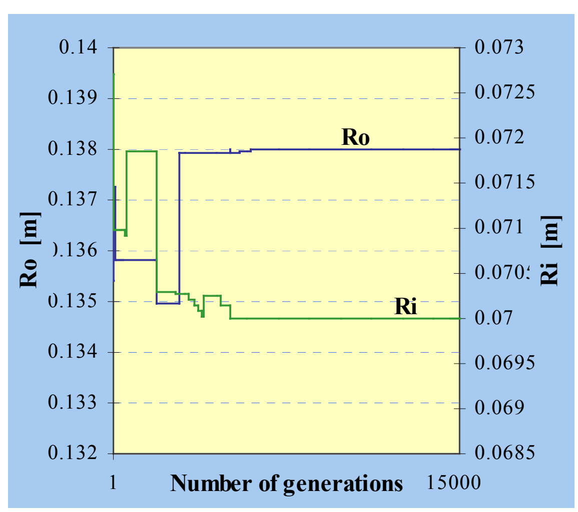


Fig. 4. Variables value change during generations

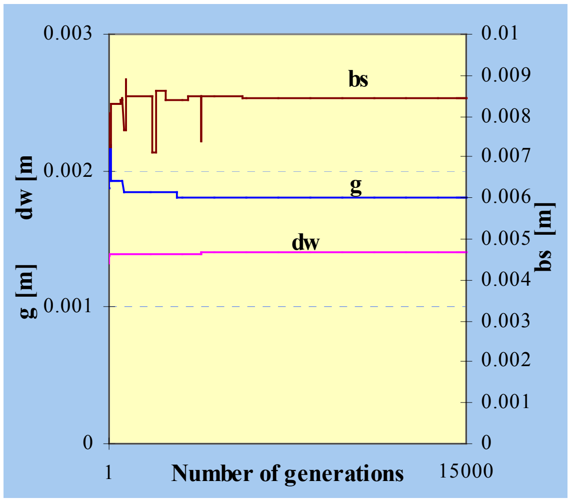


Fig. 5. Variables value change during generations

In order to be able to analyse the two models of the PMDM accurately, a calculation of the magnetic field has to be performed for all models.

## 4. Structural modelling of the PMDM for FEM analysis

The quasi 3D method which is adopted for this analysis [5] consists of a 2D FEM calculation of the magnetic field in a three dimensional radial domain of the axial field motor. For this purpose, a notional radial cut through the two stators and one rotor of the disc motor is performed and then opened out into linear form, as shown in Fig.6 and Fig.7, respectively. By using this linear quasi three dimensional model of the disc motor, which is divided into five segments, as presented in Fig.8, it is possible to model the skewing of the magnets and also to simulate the vertical displacement and rotation of the rotor. Due to the symmetry of the machine the calculation of the motor is performed only for one quarter of the permanent magnet disc motor or for one pair of permanent magnets.

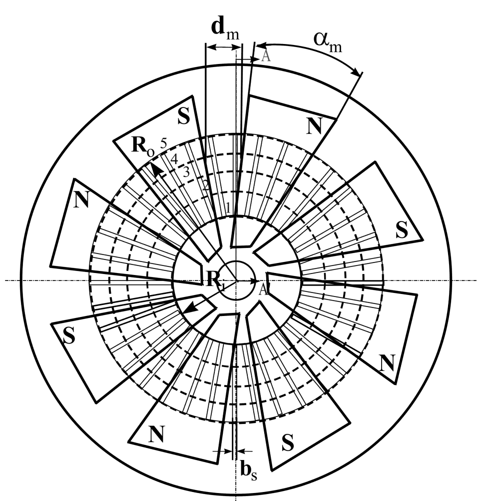


Fig. 6. Radial division of the motor into 5 segments

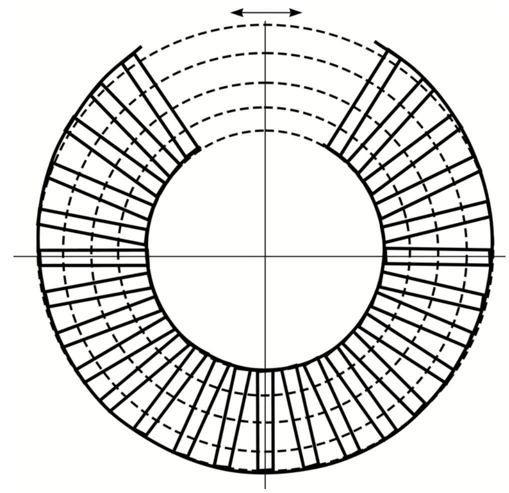

Fig. 7. Radial cut of the motor

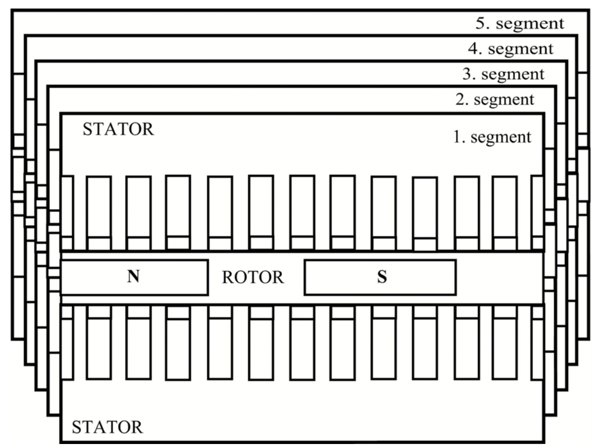


Fig. 8. Linear presentation of the motor

## 5. Magnetic field calculation

After the permanent magnet disc motor has been properly modelled, in the processor mode a quasi-3D FEM magnetic field calculation of the two models of motors for each segment separately at no load and at rated current load is performed. The distribution of the magnetic field for the two models of the motor for the middle segment at no load and at rated current load is presented in Fig. 9, Fig. 10, Fig. 11 and Fig. 12, respectively.

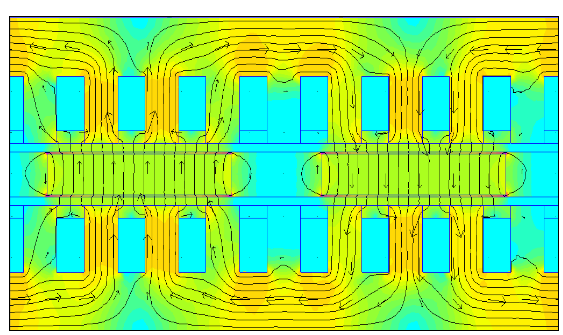

Fig. 9. Magnetic field distribution for the prototype in the middle segment at no load

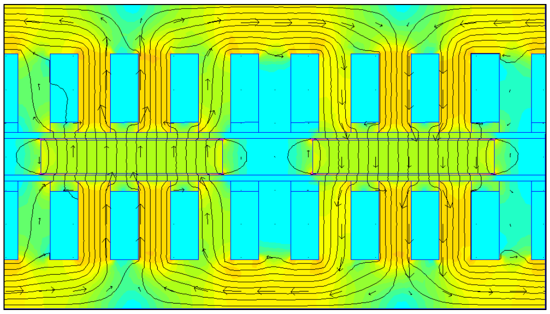

Fig. 10. Magnetic field distribution for the optimised model in the middle segment at no load

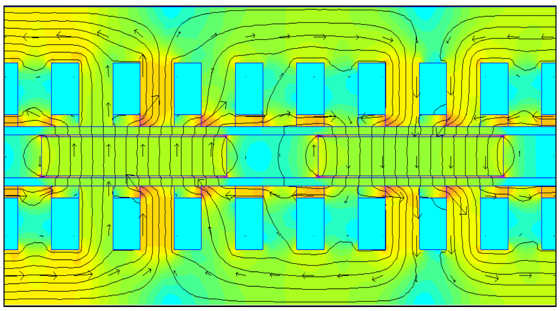

Fig. 11. Magnetic field distribution for the prototype in the middle segment at rated load

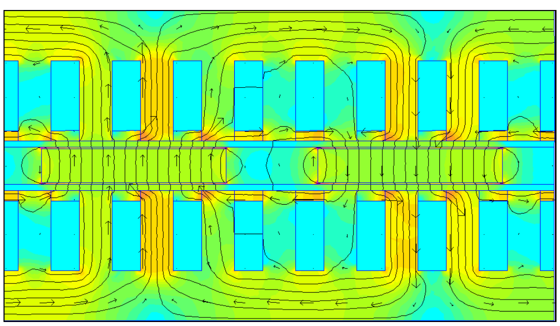

Fig. 12. Magnetic field distribution for the optimised model in the middle segment at rated load

In the postprocessor mode of the program [6] using the data from the magnetic field calculation, the value of the air gap flux density in the middle of the air gap can be calculated by using equation (13) and solving it numerically.

$$\mathbf{B} = \text{curl } \mathbf{A} \tag{13}$$

The values of the air gap flux density for the two models of the permanent magnet disc motor are calculated for different current loads and for all five segments. Due to lack of space the distribution of the air gap flux density is only presented for the middle segment of the permanent magnet disc motor models at no load and at rated current load as shown in Fig. 13, Fig. 14, Fig. 15, and Fig. 16, respectively.

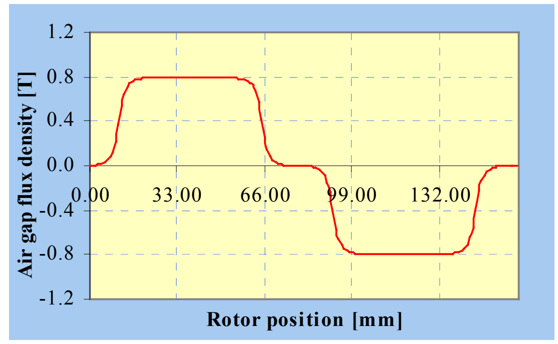


Fig. 13. Air gap flux density distribution for the prototype in the middle segment at no load

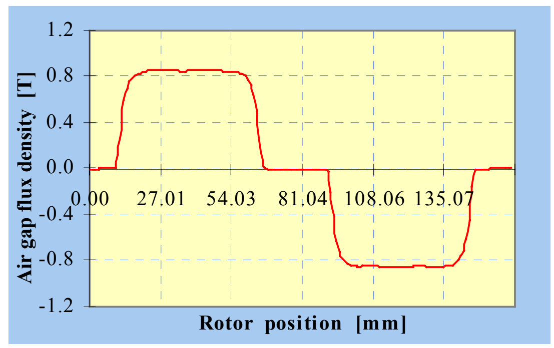


Fig. 14. Air gap flux density distribution for the optimised model in the middle segment at no load

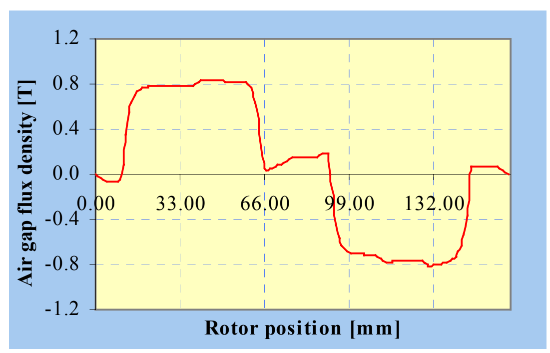


Fig. 15. Air gap flux density distribution for the prototype in the middle segment at rated load

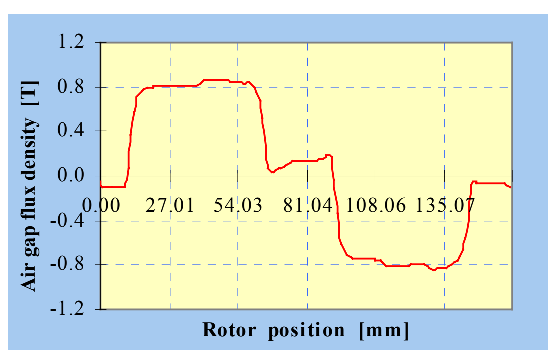


Fig. 16. Air gap flux density distribution for the optimised model in the middle segment at rated load

It is evident that the GA optimised solution in relation to the prototype has an improved magnetic field distribution in all segments of the magnetic circuit and higher air gap flux density.

## 6. Comparative analysis of the two PMDM models

From the presented values in Table IV it is evident that there is a change in the value of the air gap flux, air gap flux density, phase current, phase resistance, ohmic losses, iron losses and consequently change of the value of the efficiency of the optimised model in relation to the prototype. The reason for the flux and flux density change in the optimised model in relation to the prototype is the change of the dimensions of the stator and therefore the change in the magnetic field distribution. The change in the values of the flux density in different sections of the stators also results in difference in the iron losses, as it is shown in Table IV. The reduction of the ohmic and iron losses in the optimised model in relation to the prototype results in improvement in the value of the efficiency of the motor. This is a very important improvement for the overall performance of the motor since this motor is going to be used as a motor drive in an electric vehicle supplied by rechargeable batteries. All the above mentioned changes could also influence the transient and steady state performance of the motor, which is going to be investigated in the near future in order to have a whole picture of the overall performance of the optimised model in relation to the prototype.

Table IV. Prototype and GA solution data comparison

| Parameters | Prototype | GA solution |
|---|---|---|
| Air gap flux $\Phi_g$ [Wb] | 0.00256 | 0.00265 |
| Air gap flux density $B_g$ [T] | 0.650 | 0.671 |
| Phase current $I$ [A] | 8.723 | 8.884 |
| Phase resistance $R$ [Ω] | 1.245 | 0.563 |
| Ohmic losses $P_{Cu}$ [W] | 345.43 | 162.24 |
| Iron losses $P_{Fe}$ [W] | 15.038 | 19.61 |
| Efficiency | 0.8319 | 0.8621 |

The parameter values of the optimised model in relation to the prototype values are presented in Fig. 17. The values of the optimised model are presented in percents in relation to the prototype values which are all equal to 100 percent.

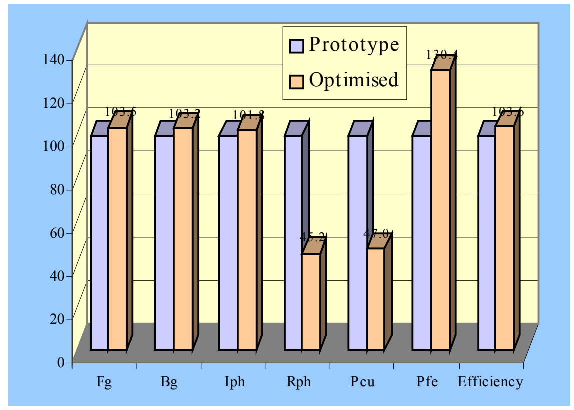


Fig. 17. Comparative data of optimised model in relation to the prototype motor presented in percents

## 7. Conclusion

At the beginning of this paper the investigated motor model and its mathematical are presented. A brief presentation of the genetic algorithm and its main genetic operators implemented in the optimisation program GA-ODEM is also presented. According to the investigation and the presented results, it can be concluded that the GA is a very suitable tool for design optimisation of PMDM and electric machines in general. By using GA the risk of trapping in a local maximum or minimum is extremely reduced, which is very difficult to eliminate in deterministic methods. At the end the quality of the GA optimised model has been proven through the FEM data analysis of the prototype and optimised solution.

## References


[1] Holland, H. “Adaptation in Natural and Artificial Systems,” Ann Arbor, Univ. of Michigan Press, 1995.

[2] G. Cvetkovski, L. Petkovska, M. Cundev and S. Gair, "Mathematical model of a permanent magnet axial field synchronous motor for GA optimisation", in Proc. ICEM'98, Vol. 2/3, pp. 1172-1177.

[3] G. Cvetkovski, L. Petkovska, M. Cundev, “Evaluation of Methods for PM Disc Motor Optimal Design”, in Conf. record of the 15th ICEM2002 on CD, p.p. 1-6.

[4] G. Cvetkovski, L. Petkovska, M. Cundev and S. Gair, "GA approach in design optimisation of a PM disk motor", COMPEL, Vol. 19, No. 2, 2000, pp. 608-614.

[5] G. Cvetkovski, L. Petkovska, M. Cundev and S. Gair, "Quasi 3D FEM in function of an optimisation analysis of a PM disk motor", in Proc. ICEM2000, Vol. 4/4, pp. 1871-1875.

[6] D. Meeker, “Finite Element Method Magnetics Version 4.0”, User's Manual, University of Virginia, Charlottesville, USA, 2004.